

# Microwave fluctuational conductivity in $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$

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**Abstract.** We present measurements of the microwave complex conductivity at 23.9 and 48.2 GHz in  $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$  films, in the fluctuational region above  $T_c$ . With increasing temperature, the fluctuational excess conductivity drops much faster than the well-known calculations within the time-dependent Ginzburg-Landau theory [H. Schmidt, Z. Phys. **216**, 336 (1968)]. Approaching the transition temperature, slowing down of the fluctuations is observed. We discuss the results in terms of a modified Gaussian theory for finite-frequency fluctuational conductivity, where renormalization is introduced in order to account for the  $T \rightarrow T_c$  regime, and a spectral cutoff is inserted in order to discard high-momentum modes. The data are in excellent agreement with the modified theory, when formulated for three-dimensional, anisotropic superconductors, in the whole experimentally accessible temperature range, and for both frequencies.

**PACS.** 74.25.Nf Response to electromagnetic fields (nuclear magnetic resonance, surface impedance, etc.) – 74.40.+k Fluctuations (noise, chaos, nonequilibrium superconductivity, localization, etc.) – 74.72.Bk Y-based cuprates

## 1 Introduction

High- $T_c$  superconductors are particularly suitable for the study of fluctuational effects: high critical temperatures, short coherence lengths and strong anisotropy have as a direct consequence a strong widening of the temperature window where fluctuations dictate the behavior of various physical observables. In particular, it is widely recognized that the temperature region where critical fluctuations might be observable can be of order  $\sim 1$  K [1]. On the other hand, the widening of the fluctuational temperature window extends to the high temperature region, thus giving the opportunity to test the validity range of fluctuational models beyond the  $T \rightarrow T_c$  limit. It appears that both the regions close to and far from  $T_c$  are worth of being investigated in cuprates.

Like in conventional, low- $T_c$  superconductors [2], most of the experimental data have been collected by measuring dc conductivity [3–13]. Finite-frequency experiments, however, are a potential source for additional information, such as the estimate of the Ginzburg-Landau (GL) relaxation time. Moreover, a finite-frequency study in the fluctuational regime is in principle a more stringent test for theoretical models, since at a single frequency two curves (real and imaginary parts) have to be fitted by the model with the same parameters, and similarly the

model must fit to the frequency dependence. A drawback is that measurements at frequencies high enough to yield data significantly different from the dc case (typically, in the microwave range) are rather difficult to perform near the normal state (most of the microwave experiments are optimized to small values of the impedance, i.e. well below the transition). In fact, in low- $T_c$  superconductors only a very few measurements of the complex excess conductivity have been performed [14], most of them at microwave frequencies or above. In cuprates, some measurements focussed to the temperature region very close to  $T_c$ , in order to assess the relevance of critical fluctuations [15–18], and only a few reports [19,20] explored a wider temperature range above  $T_c$ . In this latter case, a significant reduction of the fluctuational conductivity below the theoretical expectations as calculated within the time-dependent Ginzburg-Landau theory [21] was found. This feature was connected to the progressively smaller contribution of the high-momentum modes to the excess conductivity, as the temperature is raised above  $T_c$ : the so-called short-wavelength cutoff (SWLC) regime. Such a physical phenomenon has been successfully invoked to describe the excess dc conductivity in conventional superconductors [22] and cuprates [3,5,11–13]. It is then interesting to check this approach against a more stringent test, such as multifrequency measurements of the complex conductivity.

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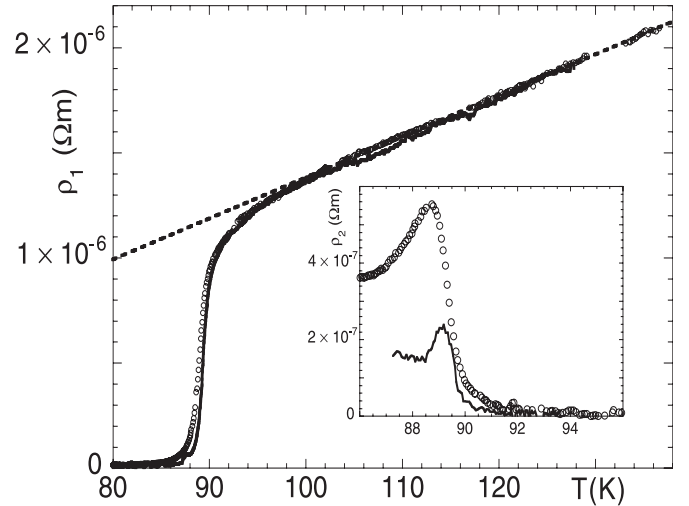
We notice that all the estimates of the excess conductivity (in zero magnetic field) rely on the assumption of some kind of normal state, existing at temperatures well above  $T_c$ , from which superconductivity emerges. In this paper we will follow the same approach, assuming in particular a linear  $T$ -dependent normal state above  $T_c$ , but we mention that alternative scenarios, connected to the so-called pseudogap phase, could affect some of the conclusions that have been given in all the papers devoted to the zero-field paraconductivity. To mention a few of the possible alternative scenarios, intrinsically inhomogeneous superconductivity [23] would lead to a dissipationless state at the percolation threshold, with excess diamagnetism and anomalous frequency dependence of the surface impedance in the pseudogap region; preformed pairs well above  $T_c$ , invoked for the interpretation of the optical conductivity [24], and competing order parameters [25], with a hidden phase transition at the pseudogap  $T^*$ , could as well increase the in-plane conductivity, thus giving an estimate of the paraconductivity lower than the one obtained with a linearly extrapolated normal state. It is then in any case necessary to consider models that predict a reduced paraconductivity with respect to the established theory [21].

In this paper we present measurements of the fluctuational complex conductivity  $\Delta\sigma$  in two high quality  $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$  (YBCO) thin films as a function of the temperature, and for two microwave frequencies (23.9 and 48.2 GHz). We show that the well-established theoretical result [21] does not describe either the frequency dependent excess conductivity approaching  $T_c$ , or the fast drop of the excess conductivity as the temperature is raised far from  $T_c$ . We extend the established model [21] to include the short-wavelength cutoff [26], which accounts for the reduced paraconductivity far from  $T_c$ , and the renormalization [17,27], effective close to  $T_c$ . The extended model describes very well the temperature and frequency dependence of the complex conductivity. Several superconducting parameters are also evaluated.

## 2 Experimental section

The complex conductivity was obtained from measurements of the temperature dependence of the quality factor and frequency shift of cylindrical metal cavities, resonant in the  $\text{TE}_{011}$  mode. The sample was mounted as an end wall, so that microwave currents flow in the  $(a, b)$  planes, and measurements yielded the  $(a, b)$  plane response. The experimental systems, extensively described elsewhere [28–30], were modified in order to measure the frequency shift [31]. Two cavities were used, for measurements at  $\omega/2\pi = 23.9$  and 48.2 GHz. Accurate calibration of the resonators was performed, in order to obtain the effective surface resistance  $R^{\text{eff}}$  and the temperature variation of the effective surface reactance  $\Delta X^{\text{eff}}$ .

Two  $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$  films, with transition temperatures  $T_c \sim 90$  K, were grown on  $1 \text{ cm} \times 1 \text{ cm} \times 1 \text{ mm}$   $\text{LaAlO}_3$  substrates by planar high oxygen pressure dc sputtering technique. The thickness of the films was



**Fig. 1.** Measurements of the real part of the resistivity at 48.2 GHz (open symbols) and 23.9 GHz (thick line). Dashed line: extrapolated normal-state resistivity. Inset: imaginary part of the resistivity. Symbols as in main panel. Measurements taken on sample YA.

$d = (2200 \pm 200) \text{ \AA}$ . Details of the film growth [32] and characterization [32,33] have been reported elsewhere. Typical characteristics of these films are [32]  $\Delta T_c < 0.5$  K (from inductive measurements), surface resistance below  $80 \mu\Omega$  at 77 K and 5.4 GHz, and FWHM of the rocking curves of the (005) peak of  $0.1^\circ$  (indicating excellent  $c$ -axis orientation). X-ray  $\Phi$ -scan analysis [33] indicated strong biepitaxiality along  $(a-b)$  plane. Average roughness of  $20 \text{ \AA}$  over  $1 \mu\text{m} \times 1 \mu\text{m}$  area was determined by AFM. The thickness of the films was of the order of the zero-temperature London penetration depth, so that close and above  $T_c$  the thin-film approximation is fully justified. Thus, the measured effective impedance directly yields the complex resistivity [34] through:  $R^{\text{eff}} \simeq \rho_1/d$ ,  $\Delta X^{\text{eff}} \simeq \Delta\rho_2/d$ . By accurate comparison with the cavity background, we did not resolve a frequency shift due to the sample above  $\sim 95$  K. We then assumed  $\rho_2 = 0$  in our near-optimal doping samples in the normal state (recent microwave measurements [35] have shown that this *ansatz* can be violated in underdoped samples, but it is valid in optimally doped cuprates), so that from the measured quality factor and frequency shift we get the complex resistivity,  $\rho(T) = \rho_1(T, \omega) + i\rho_2(T, \omega)$ . The as-extracted complex resistivity of sample YA is reported in Figure 1 for the two frequencies investigated.

The complex excess conductivity  $\Delta\sigma(T, \omega) = \Delta\sigma_1(T, \omega) + i\Delta\sigma_2(T, \omega)$  was calculated by subtracting the temperature dependent normal state real resistivity  $\rho_n(T) = 1/\sigma_n(T)$ , linearly extrapolated above 130 K, accordingly to the expression:

$$\rho(T, \omega) = \rho_1(T, \omega) + i\rho_2(T, \omega) = \frac{1}{\sigma_n(T) + \Delta\sigma(T, \omega)}. \quad (1)$$

We carefully checked that by changing the temperature range of the linear fit of the normal state resistivity in the

region 125 K–165 K we did not affect the obtained  $\Delta\sigma$  below 110 K.

In doing all the necessary transformations from measured quality factor and frequency shift to complex excess conductivity, an easily calculated geometrical factor is needed. Errors in the geometrical factor, as well as in the estimate of  $d$ , all reflect on a simple scale factor in the calculated conductivity, without affecting the temperature dependence.

Before discussing the data for  $\Delta\sigma$ , we summarize the theoretical framework that we adopt.

### 3 Theoretical background

In this section we summarize and further extend well known and recent results for the complex excess conductivity of an anisotropic, three dimensional superconductor. Explicit calculations, in 3D, 2D and 1D, have been reported previously [26].

On theoretical grounds, fluctuational phenomena in superconductors have been addressed by microscopic as well as by phenomenological theories (see, e.g., the reviews in Refs. [2, 36, 37]). The time-dependent Ginzburg-Landau (TDGL) approach is very useful since it allows for a first determination of important parameters, such as the coherence length and the relaxation time. Moreover, it often captures, in its simplicity, the basic ingredients of the underlying physics. In this section, we will give a formulation of the finite-frequency fluctuational excess conductivity in terms of this formalism. Since we are dealing with YBCO, anisotropy has to be taken into account. Due to the moderate anisotropy of this compound, we choose a description in terms of uniaxially anisotropic three-dimensional superconductor (we will come back to the consistency of this choice in the Discussion).

Calculation of fluctuational conductivity can be performed within several formalisms, such as the Kubo formula, the Boltzmann equation [38, 39] and the correlation-function formalism. We adopt here the model based on the correlation-function formalism [27], extended to include the uniaxial anisotropy [17]. This approach is based on the following very general points:

- 1- The GL functional is written as an expansion in  $\psi$  (the order parameter).
- 2- The response to a time-varying field  $\mathbf{A}(t)$  ( $\mathbf{A}$  is the vector potential) is determined by the current operator averaged with respect to the noise (represented below by the brackets), and it can be expressed as a function of the correlation function of the order parameter  $C(\mathbf{r}, t; \mathbf{r}', t') = \langle \psi(\mathbf{r}, t) \psi^*(\mathbf{r}', t') \rangle$  at equal times:

$$\langle J_x(t) \rangle = -\frac{\hbar e^*}{m_x} \int \frac{d^3q}{(2\pi)^3} q_x C \left[ \mathbf{k} = \mathbf{q} - \frac{e^*}{\hbar} \mathbf{A}(t); t, t \right] \quad (2)$$

where the momentum dependence has been shifted from  $\mathbf{k}$  to the new vector  $\mathbf{q} = \mathbf{k} + (e^*/\hbar) \mathbf{A}(t)$ ,  $e^* = 2e$  is twice the electronic charge, and  $m_j$  are the masses of the pair along the main crystallographic directions.

Standard calculations of the fluctuation conductivity are performed using the following conditions:

- i) integration can be extended to the full momentum space, being the contribution at  $q = 0$  the most diverging close to  $T_c$ ;
- ii) terms containing powers of  $\psi$  higher than 2 are dropped in the GL functional.

Using these hypotheses, one gets in a uniaxially anisotropic 3D superconductor, and in absence of a dc magnetic field, the in-plane excess conductivity (in Système International units):  $\Delta\sigma_\infty(\epsilon, \omega) = \frac{e^2}{32\hbar\xi_c(0)\epsilon^{1/2}} [S_+(w) + iS_-(w)]$ , where  $\epsilon = \ln(T/T_c)$  is the reduced temperature [40],  $\xi_c(0)$  is the out-of-plane zero-temperature GL coherence length and  $w = \omega\tau$  with  $\tau = \tau_0/\epsilon$  the temperature dependent GL relaxation time.  $S_+(w)$  and  $S_-(w)$  are the scaling functions as can be found in reference [27], and they have the property that  $S_+(w \rightarrow 0) \simeq 1 - \frac{w^2}{16}$  and  $S_-(w \rightarrow 0) \simeq \frac{w}{6}$ . We will refer to this result as “infinite cutoff approximation” (IC), and we will use the subscript “ $\infty$ ” for it. The above reported expression for  $\Delta\sigma_\infty$ , originally derived [21] for isotropic superconductors, was found to well describe microwave fluctuational conductivity in conventional superconductors [14]. Moreover, a scaling property was found close to  $T_c$  in YBCO films in swept-frequency measurements [16], even if the temperature dependence of  $\tau$  was markedly different from the GL prediction.

At temperatures sufficiently higher than  $T_c$  the approximation i) above should be somehow relaxed. In fact, equation (2) should be integrated only over momenta that give a physical contribution: modes with  $q > \xi_0^{-1}$ , where  $\xi_0$  is the temperature-independent coherence length, should be discarded. This fact was recognized early in the study of the dc excess conductivity [22], and largely applied to various superconductors [9, 11–13, 22, 41]. However, the absence of explicit expressions for the complex excess conductivity has limited the application of such approach to a few experimental results, and numerical methods had to be used [19, 20]. The usual way to discard high-momentum modes is to introduce a spectral cutoff  $q_j^{max} = \Lambda_j \xi_{j0}^{-1}$  for each crystallographic direction in equation (2) and to calculate the cutoff excess conductivity. In order to simplify the resulting expression, and to reduce the number of fitting parameters, we choose a single cutoff number imposing:  $\sqrt{\sum_{j=1,\dots,3} [q_j^{max} \xi_j(0)]^2} < \Lambda$  (Ref. [42]). The resulting complex excess conductivity can be calculated explicitly [26], and for a uniaxial superconductor the result reads:

$$\Delta\sigma_{3D} = \frac{e^2}{32\hbar\xi_c(0)\epsilon^{1/2}} \frac{16}{3\pi w^2} \left\{ \text{atn } K - (1 - iw)^{3/2} \text{atn} \left( \frac{K}{\sqrt{1 - iw}} \right) + iw \left[ \frac{K}{2(1 + K^2)} - \frac{3}{2} \text{atn } K \right] \right\} \quad (3)$$

where  $K = \Lambda/\epsilon^{1/2}$ . In order to clarify some of the main features of equation (3), it is useful to introduce the small  $w$

expansion. Up to terms of order  $w^2$  one has:

$$\Delta\sigma_1(\epsilon, w \ll 1) \simeq \frac{e^2}{32\hbar\xi_c(0)\epsilon^{1/2}} \times \left\{ \frac{2}{\pi} \left[ \text{atn}K - \frac{K}{(1+K^2)^2} \left( 1 + \frac{5}{3}K^2 \right) \right] + o(w^2) \right\} \quad (4)$$

$$\Delta\sigma_2(\epsilon, w \ll 1) \simeq \frac{e^2}{32\hbar\xi_c(0)\epsilon^{1/2}} \frac{w}{6} \frac{2}{\pi} \left[ \text{atn}K + \frac{K}{(1+K^2)^3} \left( K^4 - \frac{8}{3}K^2 - 1 \right) \right]. \quad (5)$$

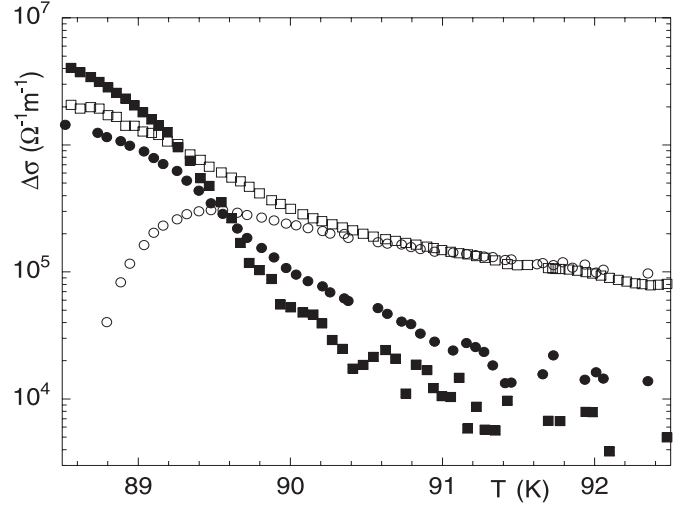
Like in the IC approximation, the leading term in the real part is frequency independent and the corresponding term in the imaginary part is proportional to the frequency. On the other hand, the cutoff affects strongly the temperature dependences, in particular on the real part (see also the discussion in Ref. [26]). It is then expected that the analysis of the data in the region  $w \ll 1$  can be used to easily discriminate between the IC and the finite-cutoff expressions.

Regarding the assumption ii), it becomes less and less acceptable approaching very closely  $T_c$ . In order to take into account the critical fluctuation regime, Dorsey [27] has released that assumption by using the Hartree approximation for the quartic term in  $\psi$ , and keeping the validity of i) above. The resulting expression of the complex excess conductivity remained unaffected, including the scaling functions  $S_+(x)$  and  $S_-(x)$ , and the effect of the extended approach was to change the temperature dependence of the reduced temperature  $\epsilon$  (renormalization) according to:

$$\tilde{\epsilon} = \epsilon \frac{\epsilon}{\eta} \left[ 1 + \left( 1 + \frac{\epsilon}{\eta} \right)^{1/2} \right]^{-2}. \quad (6)$$

When one takes into account the anisotropy [17], in the preceding expression  $\eta = l^2\kappa^4\gamma^2\xi_{ab}^2(0)$ ,  $l = (e^2\mu_0 k_B T / 4\pi\hbar^2)$ ,  $\mu_0$  is the magnetic permeability,  $\gamma = \xi_{ab}(0)/\xi_c(0)$  is the anisotropy factor and  $\kappa = \lambda_{ab}(0)/\xi_{ab}(0)$  is the Ginzburg parameter, and  $\xi_c(0)$  and  $\lambda_{ab}(0)$  are the zero-temperature values of the GL out of plane coherence length and in-plane penetration depth, respectively. Summarizing, in the Hartree approximation the fluctuational excess conductivity has the same form as in the IC, unrenormalized limit, with the replacement  $\epsilon \rightarrow \tilde{\epsilon}$ . We remark that close to  $T_c$  the temperature dependence of the GL relaxation time changes to  $\tau \sim \epsilon^{-2}$ , showing the so-called slowing down of fluctuations.

We note that the two above-described modifications to the IC, unrenormalized approximation have a significant relevance in different temperature ranges: the renormalization is relevant only close to  $T_c$  ( $T - T_c \lesssim 2$  K), while the influence of the cutoff in equation (3) shows up above that temperature range. We then cast together the two modifications by substituting  $\epsilon$  with  $\tilde{\epsilon}$  in equations (3, 4, 5).



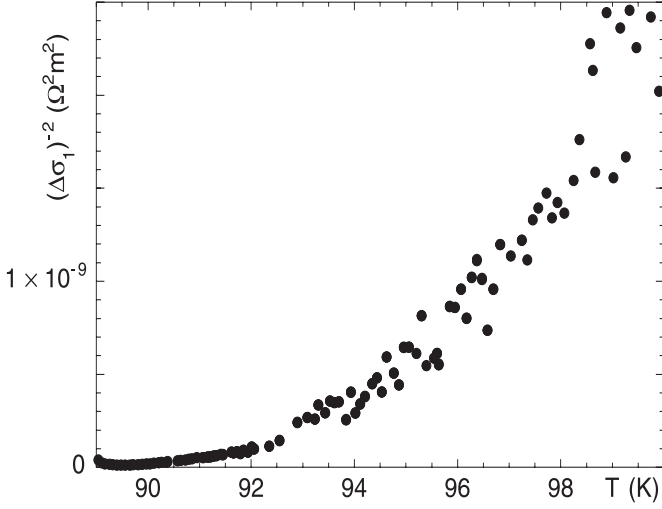
**Fig. 2.** Complex excess conductivity as a function of the temperature in sample YA. Open symbols:  $\Delta\sigma_1$ , full symbols:  $\Delta\sigma_2$ . Squares: 23.9 GHz, circles, 48.2 GHz. Above  $\sim 90.5$  K,  $\Delta\sigma_1$  does not exhibit significant frequency dependence. It is also seen that  $\Delta\sigma_2$  scales approximately as  $\omega$ .

In concluding this section, we ought to mention that the cutoff approach is still somehow debated. However, it is believed to be a fundamental aspect of the GL theory [37,43,44]. In the specific case of the dc, 2D excess conductivity, we have thoroughly discussed the similarities between the GL cutoffted model and the microscopic theory [45], and we have shown that the GL expression with cutoff reproduces nearly exactly the microscopic theory for clean superconductors, when the appropriate cutoff number is chosen.

Finally, we comment on the fact that the momentum cutoff is not the only possible approach in order to extend the validity of the GL theory beyond the vicinity of  $T_c$ . In fact, an alternative choice is the so-called “total energy” cutoff [12], where it is the kinetic + localization energy of the (fluctuating) Cooper pair that is limited to some value. The results for the ac excess conductivity are formally identical to the ones reported in Section 3, with the substitution  $\Lambda \rightarrow \sqrt{c^2 - \epsilon}$ , with  $c$  a new cutoff number, as noted earlier [26], in analogy with the other physical quantities [46]. Equation (3) modified in order to use a total energy cutoff would exhibit a stronger decrease of the excess conductivity at high  $\epsilon$ , with vanishing  $\Delta\sigma$  at  $\epsilon = c^2$ .

## 4 Discussion

We now discuss our data in light of the theoretical frame of Section 3. We first identify the temperature region where  $w \ll 1$  in our data. In Figure 2 we present the complex excess conductivity at 23.9 and 48.2 GHz in sample YA as a function of  $T$ . As can be seen, at temperatures above 90.5 K the frequency dependence is nearly absent in  $\Delta\sigma_1$ , indicating that the GL relaxation rate  $1/\tau_0$  is much larger than our measuring frequency (see Eq. (4)).



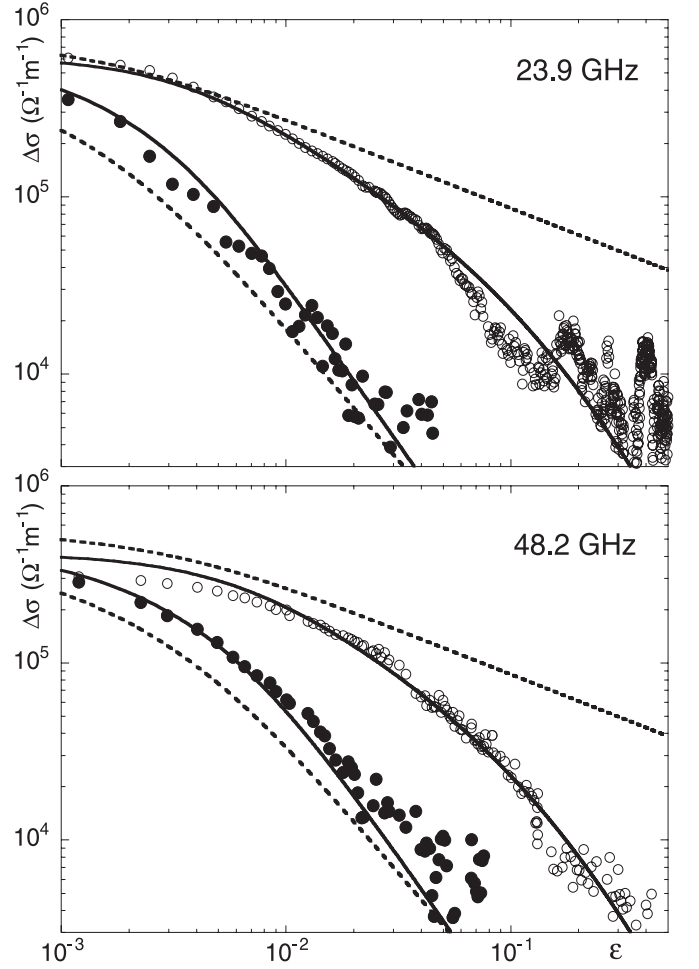
**Fig. 3.** Comparison with plain 3D infinite cutoff predictions,  $(\Delta\sigma_1)^{-2} \propto T - T_c$ . Clear curvature in  $(\Delta\sigma_1)^{-2}$  is evident, in disagreement with the infinite cutoff approximation. Data refer to sample YA at 48.2 GHz.

Consistently (see Eq. (5)), the imaginary part  $\Delta\sigma_2$  scales approximately as the ratio of the measuring frequencies. These features represent the experimental indication that, for most of the temperature range here explored, the small frequency regime is reached.

We now show that the simple IC model is not compatible with our data. Using the fact that, within 1%, one has  $\epsilon \simeq \frac{T}{T_c} - 1$  up to  $T = 1.1T_c$ , the IC approximation in the small  $w$  regime predicts  $\Delta\sigma_{1,\infty}^{-2} \propto T - T_c$  in a rather wide temperature range, as opposed to the cutoff expression (see Eq. (4)), that removes this simple temperature dependence.

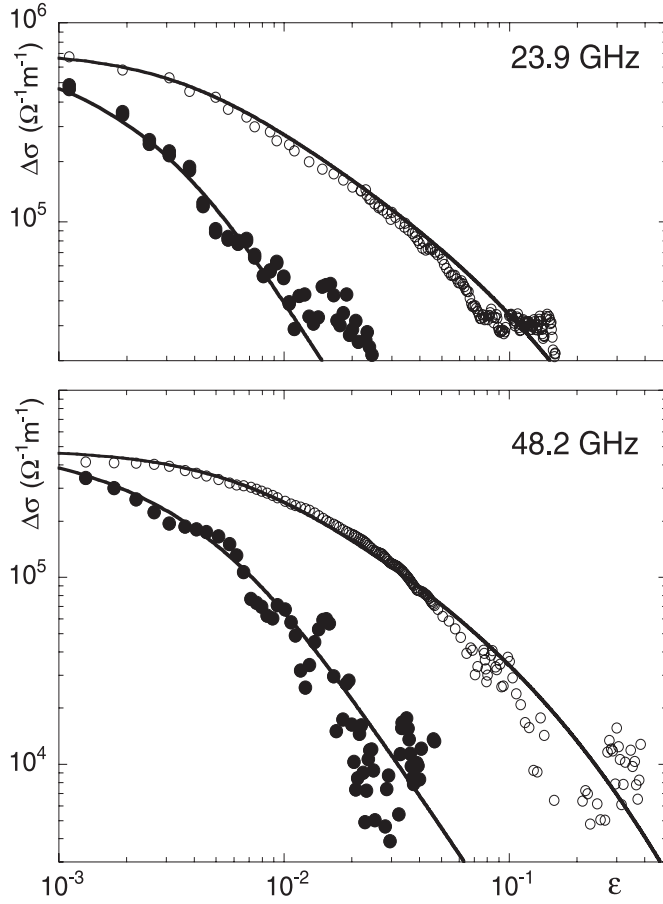
In Figure 3 we plot the data as  $(\Delta\sigma_1)^{-2}$  vs.  $T$ . It is clear that our data do not follow a linear law in a significant temperature range, as would be expected from the IC approximation. The temperature and frequency dependence of  $\Delta\sigma$  further indicate (see later and Fig. 4) that the simple IC prediction does not describe our data. We conclude that a proper description for our data of the fluctuational conductivity must be found beyond the IC approximation.

We adopt the extension of the TDGL theory described in Section 3. In order to take into account temperatures far from  $T_c$  as well as the approach to the critical region, we use in the full expression, equation (3), the renormalized reduced temperature, equation (6). The resulting explicit expression for the complex  $\Delta\sigma$  contains as parameters the cutoff number  $\Lambda$  (effective mostly at high temperatures), the out-of-plane, zero-temperature GL coherence length  $\xi_c(0)$  (which acts mainly as a scale factor), the GL relaxation time  $\tau_0$  (determined mainly through  $\Delta\sigma_2$ ), the number  $\eta$  (which determines the reduced temperature range of the renormalized regime), or alternatively the zero-temperature, in-plane penetration depth  $\lambda_{ab}(0)$ , and  $T_c$ .



**Fig. 4.** Complex excess conductivity as a function of the reduced temperature  $\epsilon = \ln(T/T_c)$ . Open symbols:  $\Delta\sigma_1$ , full symbols:  $\Delta\sigma_2$ . Upper panel: 23.9 GHz, lower panel: 48.2 GHz. Data taken on sample YA. Full lines are fits with equation (3), with parameters as in Table 1. Dashed lines are unrenormalized, infinite cutoff, best fits, with  $\tau_0 = 27$  fs and the scale factor  $\xi_c(0) = 2.8$  Å chosen to match the experimental  $\Delta\sigma_1$  at 23.9 GHz. The conventional model is clearly inadequate in order to simultaneously fit the four experimental curves.

An extremely precise determination of  $T_c$  is not essential, since the main point of this paper is the “high” temperature regime. Nevertheless, scaling theories [27] in a 3D superconductor suggest the following path for its evaluation: in fact, one has  $\Delta\sigma_1(T \rightarrow T_c) = \Delta\sigma_2(T \rightarrow T_c)$ , so that the crossing point of the real and imaginary parts should give the critical temperature. Unfortunately this determination can be easily affected by experimental errors in the evaluation of the absolute value of the excess conductivity such as those mentioned in Section 2. We have then chosen to keep  $T_c$  as a parameter of the fit, keeping in mind that it cannot be noticeably different from the cross-point  $T^\times$  of the real and imaginary parts of  $\Delta\sigma$ . The values of the crossing temperature  $T^\times$  and the fitted  $T_c$  are reported in Table 1, and good agreement is found.



**Fig. 5.** As in Figure 4, but data are taken on sample YB. Open symbols:  $\Delta\sigma_1$ , full symbols:  $\Delta\sigma_2$ . Upper panel: 48.2 GHz, lower panel: 23.9 GHz. Full lines are fits with equation (3), with parameters as in Table 1.

All the parameters are fixed by the simultaneous fitting of four experimental curves in each sample: real and imaginary parts at two different measuring frequencies.

Fits are presented in Figures 4 and 5 as continuous lines. It is clear that the extended model well describes our data for both microwave frequencies, as opposed to the failure of the IC approximation (dashed lines in Fig. 4). In particular, it is the temperature dependence at large  $\epsilon$  that clarifies the role of the cutoff, while the slowing down of fluctuations at  $T_c$  is described by the renormalization.

The so-obtained fitting parameters are summarized in Table 1. The zero-temperature  $c$ -axis coherence lengths,  $\xi_c(0)$ , are in agreement with commonly accepted values [47]. The renormalization parameter  $\eta \simeq 7 \times 10^{-3}$  (corresponding to a GL  $\lambda_{ab}(0) \simeq 900$  Å) indicates that in a region  $\sim 0.6$  K wide close to  $T_c$  the corrections due to the  $\psi^4$  term cannot be neglected. The cutoff number  $\Lambda$  is of the correct order of magnitude, and appears to be sample-dependent. We already found [13] (from dc conductivity) that in slightly overdoped  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+x}$  crystals there was a doping-dependence of the cutoff number. It would be interesting to look for a similar dependence in YBCO. We note that the obtained parameters are

**Table 1.** Crossing temperatures of real and imaginary parts  $T^\times$  at the two measuring frequencies (see text) and fit parameters: critical temperatures  $T_c$ , zero-temperature  $c$ -axis coherence length  $\xi_c(0)$ , GL relaxation time  $\tau_0$ , renormalization parameter  $\eta$  (see Eq. (6)), and cutoff number  $\Lambda$ .

	Sample YA	Sample YB
$T_{23.9}^\times$ (K)	89.3	89.25
$T_{48.2}^\times$ (K)	89.5	89.3
$T_c$ (K)	89.45	89.25
$\xi_c(0)$ (Å)	3.3	2.8
$\tau_0$ (fs)	27	29
$\eta$ (Å)	$7 \times 10^{-3}$	$7 \times 10^{-3}$
$\Lambda$	0.55	0.65

consistent with the 3D treatment here employed: in fact, in order to neglect the layered structure one needs the anisotropy parameter  $\left(\frac{2\xi_c}{t}\right)^2$  ( $t \simeq 7$  Å is the spacing between superconducting planes in YBCO) to be sufficiently larger than  $\Lambda^2$  (Refs. [37, 39]). With our parameters we obtain a factor  $\sim 3$  between the two quantities, that points to an essentially 3D nature of YBCO.

The GL relaxation time is found to be slightly sample dependent, in agreement with results obtained from the analysis at microwave [16] and radio [48] frequencies and from combined excess diamagnetism and conductivity measurements [49]. The numerical value for  $\tau_0$  is noticeably larger than the BCS estimate,  $\tau_0 = \frac{\hbar\pi}{16k_B T} \simeq 17$  fs (having taken  $T = T_c$ ), in qualitative agreement with reference [16]. A possible interpretation of this finding comes from the unconventional nature of the pairing in cuprates. In fact, it is expected [37] that in a  $d$ -wave superconductor the GL relaxation time is larger than the BCS,  $s$ -wave estimate. It is clear that this intriguing point deserves further experimental work in the future, possibly expanding the frequency range explored.

## 5 Conclusions

We have measured the excess complex conductivity at two microwave frequencies in very high quality YBCO films. The temperature dependence of the data in the region  $\omega\tau \ll 1$  directly shows that the established results of the time-dependent Ginzburg Landau theory within the IC approximation do not describe our data. We have incorporated in the TDGL theory the renormalization (R) of the reduced temperature and the short-wavelength cutoff (C), obtaining a single explicit expression for the finite-frequency excess conductivity. With these modifications, the RC-GL theory describes in full our data, in the temperature and in the frequency dependence. With a reduced parameter set we are able to fit at once four experimental

curves for each sample. The very good fitting and remarkable consistency of the parameters support the idea that the proposed modifications of the TDGL model capture the essential features of the complex excess conductivity in YBCO. We have estimated the out-of-plane coherence length, the in-plane penetration depth and the GL relaxation time. The latter appears longer than the BCS  $s$ -wave value. This point, in qualitative agreement with the expectations for  $d$ -wave superconductors, deserves further experimental work in the future.

– Note added. After completing this manuscript, a related work appeared, discussing a possible effect of the cutoff close to  $T_c$  within a model for the ac fluctuation conductivity similar to the one here presented [50]. Such effect should manifest itself in an increasing  $T^\times$  with the frequency. This is qualitatively consistent with our data, see Table 1.

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- As a consequence, the cutoff  $\Lambda=0.74$  means a momentum cutoff  $Q = \xi_0^{-1}$ . We also note that our choice of the cutoff would correspond to a kinetic energy cutoff  $E_{max} < \Lambda^2 \frac{\hbar^2}{2m^* \xi^2(0)}$
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